Scaling Factor Estimation Using Optimized Mass Change Strategy, Part 2: Experimental Results

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SCALING FACTOR ESTIMATION USING AN OPTIMIZED MASS CHANGE STRATEGY.

PART 2: EXPERIMENTAL RESULTS

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Abstract

The mass change method is used to estimate the scaling factors, the uncertainty is reduced when, for each mode, the frequency shift is maximized and the changes in the mode shapes are minimized, which in turn, depends on the mass change strategy chosen to modify the dynamic behavior of the structure. On the other hand, the aforementioned objectives are difficult to achieve for all modes simultaneously. Thus, a study of the number, magnitude and location of the masses must be performed previously to the modal tests. In this paper, the mass change method was applied to estimate the scaling factors of a steel cantilever beam. The effect of the mass change strategy was experimentally studied by performing several modal tests in which the magnitude, the location and the number of the attached masses were changed.

1 Introduction

It is well known that the mode shapes can not be scaled in natural input modal analysis so that an additional method is needed to estimate the scaling factors [1]. In the past last years, some procedures have been proposed to determine the scaling factors based on the mass change method [2, 3, 4, 5]. This method consists of modifying the dynamic behaviour of the structure attaching masses to the points of the structure where the mode shapes are known. The scaling factors are estimated using the modal parameters of both the original and the modified structure.

When the mass change method is applied, a minimum frequency shift has to be achieved [3, 5, 6] in order to minimize the effect of the uncertainties on the natural frequencies and the mode shape coordinates. On the contrary, the frequency shift should not be too high in order to minimize the changes in mode shapes [4, 5, 6].

The frequency shifts and the changes in mode shapes are not only controlled by the magnitude of the masses attached to the structure but also by the number and location of the masses. This means that the mass change strategy must be studied before modifying the dynamic behaviour of the structure attaching masses [6].

In this paper, the mass change strategy proposed in [6] is studied and validated. A steel cantilever beam was used to perform the tests. The effect of the mass change strategy was experimentally studied by performing several modal tests in which the magnitude, the location and the number of the attached masses were changed. Different mass change configurations were used to modify the mass of the structure. Only the first five modes were considered in the investigation.

2 Structure

The structure tested was a steel cantilever beam as shown in Figure 1. The beam was 1875 mm length, with a rectangular 100 x 40 mm hollow profile, 4 mm thick.

The measurements were recorded by means of 8 accelerometers located as shown in Figure 1. The lumped masses needed for modifying the dynamic behaviour of the structure were attached in DOF's 1 to 7. The distance between points was 250 mm, excepting between point, 7 and 8, where the distance was reduced to 125 mm.

The natural frequencies and scaling factors, obtained from a numerical model using the properties indicated in Figure 1, are shown in Table 1.

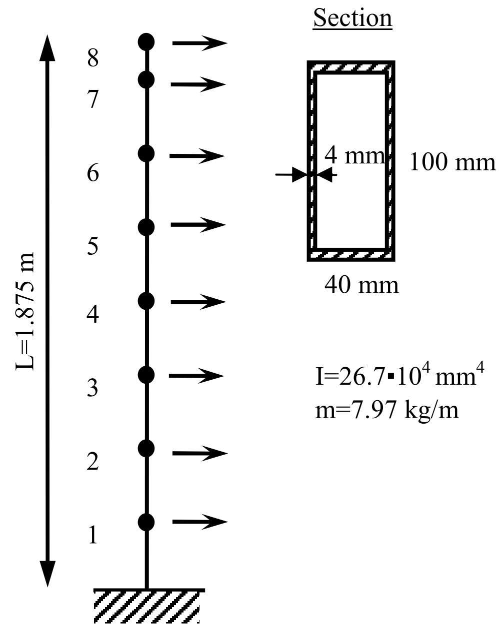

Figure 1. Structure

Table 1. Natural frequencies and scaling factors from a numerical model.

Modal parameter	Mode				
	1	2	3	4	5
Natural Frequency (Hz)	13,351	83,674	234,41	460,11	763,28
Scaling Factor	0,517	0,517	0,518	0,520	0,520

3 First operational modal testing and analysis

The first modal analysis was performed on the unmodified structure. The structure was excited moving a hand-file upward and downward along the beam so that the excitation was stationary broad banded. The responses were measured using 8 accelerometers 4508B Brüel & Kjær, located as shown in Figure 1, and recorded with a data acquisition card (National Instruments PCI4472) controlled by Labview. The tests were carried out at a sampling frequency of 3500 Hz. The natural responses were measured during a period of approximately 3 minutes.

Table 2. Natural frequencies (Hz) identified by EFDD and SSI techniques.

M ode	Natural Frequency (Hz)	
	EFDD	SSI
1	11.402	11.470
2	72.625	72.314
3	201.564	201.728
4	386.715	387.146
5	611.970	611.758

The modal analysis was performed using a natural input modal analysis software. The first five natural frequencies identified with Enhanced Frequency Domain Decomposition (EFDD) and the Stochastic Subspace Identification (SSI) are shown in table 2. The high scatter obtained in the estimated scaling factors corresponding to the first two modes, suggested to repeat the modal testing. The new tests were concentrated on the first two modes and the natural responses were measured during a period of approximately 10 minutes. The time series were decimated to a sampling frequency of 350 Hz to estimate the modal parameters of the two first modes.

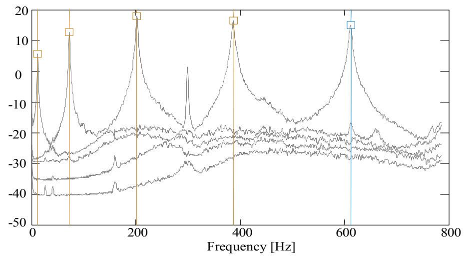

Figure 2. Modes identified for the original structure by the EFDD technique.

The estimated natural frequencies by EFDD [7] for the fist five modes are shown in figure 2.

4 Mass change strategy

In this paper, different mass change configurations (see table 4) were used to modify the dynamic behaviour of the structure in order to study the effect of the number, magnitude and location of the attached masses proposed in [6].

The first step in the strategy proposed in [6] consisted in creating a table that provides information of the contribution of a unit mass, located in the j degree of freedom, to the modification of the natural frequency corresponding to the k mode, see table 3. This table is obtained from the mode shapes of the original structure [6].

Table 3. Contribution of a unity mass to the frequency shift (in %).(see table 1 in [6])

Mode	DOF						
	1	**2**	**3**	**4**	**5**	**6**	**7**
1	0.36	2.37	8.79	20.58	39.16	64.98	100.00
2	12.17	56.33	100.00	84.43	26.33	2.09	91.25
3	48.12	100.00	28.48	14.50	76.72	24.39	42.45
4	100.0	33.82	58.53	70.78	15.95	87.73	16.81
5	100.0	13.05	74.15	54.17	28.78	95.93	2.45

Table 3 shows that if we are interested in modifying the natural frequency corresponding to the first mode, using two masses, the best locations are the DOF's 6 and 7. DOF's 3 and 7 would be the best location to modify the 2nd mode.

If we want to modify simultaneously the natural frequencies of the first two modes using two masses, we can sum the rows 1 and 2, from which is inferred that the best locations are the DOF's 4 and 7. Masses in DOF's 2 and 5 were also attached in order to validate the mass change strategy proposed in [6].

The location and magnitude, in grams, of the attached masses are shown in table 4 for each configuration. The column to the right indicates the modes that we tried to optimize simultaneously in each configuration.

The different mass change configurations can be compared creating new tables (table 5), which provide information about the relative frequency shift that we are going to obtain, compared with the frequency shift that we would obtain locating masses at positions allowing for a maximum frequency shift (maximum values of the mode shapes). In [6] was demonstrated that the frequency shifts, $\Delta\omega$ corresponding to two mass change configurations 'a' and 'b' are related by:

$$\frac{\Delta\omega_a}{\Delta\omega_b} = \frac{\{\psi_b\}^T[\Delta m]\cdot\{\psi_b\}}{\{\psi_a\}^T[\Delta m]\cdot\{\psi_a\}} \quad (1)$$

where $[\Delta m]$ is the mass change matrix and $\{\psi\}$ the unscaled mode shape.

Table 4. Different mass change configurations. Location and magnitude of the attached masses.

Nº Masses	Total Mass change (%)	DOF							
		1	2	3	4	5	6	7	Modes to be optimized
2	6.30				X (471)			X(467)	1 to 2
			X (471)			X (467)			1 to 6
3	6.18		X (310)			X (307)		X(308)	1 to 3
			X (310)				X (307)	X(308)	1 to 6
4	5.50	X (206)	X (204)			X (205)	X (206)		3 to 4
			X (204)	X (206)			X (206)	X(205)	1 to 4
5	4.85	X (144)		X (146)	X (146)		X (146)	X(144)	1 to5
		X (144)	X (144)	X (146)	X (146)		X (146)		4 to5
7	6.80	X (144)	X (146)	X (146)	X (146)	X (144)	X (144)	X(147)	1 to7

The relative frequency shift, $\Delta\omega_{47}/\Delta\omega_{25,}$ corresponding to masses in DOF's 4 and 7 and in DOF's 2 and 5, respectively, is shown in table 5. This table shows that attaching masses in the DOF's 4 and 7, the frequency shift for the first mode would be approximately 73 % of that for a maximum frequency shift, i.e., the one we would obtain attaching masses at optimal positions (6 and 7 degree of freedoms for the 1st mode). Similar comparisons can be made with other mass configurations.

Table 5 also shows that masses attached in DOF's 2 and 5 are the best option only for the 3rd mode. Masses in DOF's 4 and 7 provide higher frequency shifts for the rest of the modes.

Table 5. Predicted relative frequency shifts [6]

Mode	$\frac{\Delta\omega_{47}}{\Delta\omega_{Opt}}$	$\frac{\Delta\omega_{25}}{\Delta\omega_{Opt}}$	$\frac{\Delta\omega_{47}}{\Delta\omega_{25}}$
1	72.95	25.10	290.64
2	91.86	43.29	212.17
3	32.18	100.11	32.14
4	46.79	26.56	176.20
5	29.01	21.31	136.14

When the mass change method is used, we must also try to minimize the changes in mode shapes. This implies to minimize the terms $\frac{\omega_i^2}{\omega_t^2 - \omega_i^2}\{\psi_i\}^T[\Delta m]\cdot\{\psi_t\}$ corresponding to each mode [6]. These terms provides information of the orthogonality of the modes with respect to the matrix $[\Delta m]$, that should be minimized for each mode.

In the table 6 a comparison of the contribution of each mode to modify the mode shapes [6] is shown, for two cases with masses attached in DOF's 4 and 7 and in all DOF's (1,2,3,4,5,6, and 7), respectively. It can be seen that locating masses in all DOF's, the mass change is near to be proportional to the mass of the structure. It can be concluded that the mode shape modification using this strategy is very low compared with the configuration with masses in DOF's 4 and 7.

5 Second operational modal analysis

A second operational modal testing and analysis was performed on the modified structures. The dynamic behaviour of the structure were modified attaching lumped masses in several degrees on freedom, see table 4. The tests were carried out for the same sampling frequency used in the first modal analysis. Initially, the natural responses were measured during approximately 3 minutes. However, new modal tests concentrated on the first two modes were carried out, during approximately 10 minutes.

The natural frequencies estimated by the EFDD and SSI identification methods are presented in tables 7 and 8, respectively.

Table 6. Contribution of the mode shape modification (table 3 in [6]).

Masses in DOF's	Mode	Contribution of the mode				
		1	2	3	4	5
4 & 7	1		0.004	0.001	0.0002	0.000
	2	-0.171		0.038	-0.003	-0.002
	3	-0.157	-0.296		-0.005	-0.005
	4	-0.248	0.101	0.020		0.103
	5	-0.159	0.141	0.0545	-0.283	
All DOF's	1		-0.000	-0.000	-0.000	-0.000
	2	0.009		-0.002	-0.001	-0.000
	3	0.010	0.016		-0.003	-0.0027
	4	0.012	0.0248	0.0121		-0.0023
	5	0.021	0.012	0.0296	0.007	

Table 9 shows the experimental frequency shifts obtained with each mass change configuration are presented. Moreover, a diagonal mass matrix was assembled to predict the frequency shifts. In this case, the total mass of the cantilever beam was known so that the predictions provide good results even though the diagonal matrix is only an approximation. The frequency shifts were predicted using equation 2 [6]:

$$\frac{\Delta\omega}{\omega_0} = 1 - \sqrt{\frac{1}{1 + \frac{\{\psi_0\}^T [\Delta m] \{\psi_0\}}{\{\psi_0\}^T [m] \{\psi_0\}}}} \qquad (2)$$

Table 7. Natural Frequencies of the modified structure by EFDD.

Modes	Natural frequency (Hz)									
	Original structure	Modified structure. Masses attached in DOF								
		4 7	2 5	2 5 7	2 6 7	1 2 5 6	2 3 6 7	1 3 4 6 7	1 2 3 4 6	All DOF's
1	11.40	10.85	11.23	10.871	10.862	11.198	10.979	11.098	11.354	11.025
2	72.63	69	70.85	70.1	70.521	71.666	70.219	70.611	70.935	70.028
3	201.56	198.06	190.33	192.22	194.67	194.64	196	197.19	198.14	194.9
4	386.71	376.85	380.55	381.6	376.14	375.48	377.07	375.2	375.82	374.08
5	611.97	600.52	602.52	604.63	600.33	594.65	599.43	593.8	594.29	592.54

Table 8. Natural frequencies of the modified structure by SSI

Modes	Natural frequency (Hz)									
	Original structure	Modified structure. Masses attached in DOF								
		4 7	**2 5**	**2 5 7**	**2 6 7**	**1 2 5 6**	**2 3 6 7**	**1 3 4 6 7**	**1 2 3 4 6**	**All DOF's**
1	11.47	10.795	11.105	10.844	11.039	11.316	11.209	10.892	10.964	11.032
2	71.56	68.509	70.447	70.033	70.383	71.488	70.072	70.599	70.768	69.963
3	201.73	198.02	190.55	192.25	194.75	194.64	195.96	198.14	197.19	194.87
4	387.15	377.17	380.74	381.86	376.43	375.56	377.21	375.82	375.2	374.32
5	611.75	601.07	602.48	604.49	600.04	594.65	599.65	594.29	593.8	592.4

Table 9. Experimental and predicted frequency shifts (%) (EFDD technique)

Mode	fq. shift	Masses attached in DOF								
		4 7	**2 5**	**2 5 7**	**2 6 7**	**1 2 5 6**	**2 3 6 7**	**1 3 4 6 7**	**1 2 3 4 6**	**All DOF's**
1	Predicted	5.390	1.957	4.239	4.955	2.207	3.559	2.810	1.440	3.391
	Exper.	4.833	1.500	4.657	4.736	1.789	3.710	2.666	0.421	3.306
2	Predicted	4.840	2.371	3.233	2.803	1.232	3.094	2.565	2.269	3.277
	Exper.	4.987	2.433	3.477	2.897	1.321	3.313	2.773	2.327	3.576
3	Predicted	1.780	5.246	4.340	3.360	3.329	2.636	1.534	2.072	3.176
	Exper.	1.737	5.570	4.635	3.422	3.435	2.762	2.170	1.698	3.308
4	Predicted	2.343	1.350	1.190	2.420	2.761	2.304	2.743	2.875	3.126
	Exper.	2.550	1.593	1.324	2.735	2.906	2.495	2.977	2.818	3.266
5	Predicted	1.589	1.175	0.822	2.028	2.860	2.254	2.780	2.865	3.107
	Exper.	1.870	1.544	1.200	1.902	2.830	2.049	2.969	2.890	3.176

6 Scaling factors

The scaling factors corresponding to the first five modes of the cantilever beam studied were estimated from the modal parameters of both the unmodified and the modified structures using the equation [2, 4]:

$$\alpha_{01} = \sqrt{\frac{\left(\omega_0^2 - \omega_1^2\right)}{\omega_1^2 \cdot \{\psi_0\}^T \cdot [\Delta m] \cdot \{\psi_1\}}}, \tag{3}$$

where $\{\Psi\}$ is the unscaled mode shape, ω are the natural frequencies, α is the scaling factor, whith relates the unscaled $\{\Psi\}$ and the mass normalised $\{\phi\}$ mode shapes by $\{\phi\} = \alpha\,\{\Psi\}$, $[\Delta m]$ is the mass change matrix and the subscripts '$_0$' and '$_1$' indicates original and modified structure, respectively.

Table 10 shows the scaling factors obtained from the modal parameters estimated by the EFFD method, whereas table 11 shows the scaling factors obtained from the modal identification with the SSI method. The results correspond to mode shapes normalised to unity.

As it was indicated previously, new modal tests were carried out in order to reduce the scatter obtained in the estimated scaling factors corresponding to the first two modes. In [6] was shown that the same absolute error in the natural frequencies induces greater errors in the scaling factors at low frequencies. It is needed a good modal identification at these frequencies.

In [6] was demonstrated that the uncertainty on the scaling factor increases as the frequency shift diminishes. This means that the scaling factors obtained at low frequency shifts will show a high scatter and should be discarded.

In red color, the scaling factors estimated with frequency shifts $\Delta\omega$ less than 2% of the natural frequency of the original structure are presented. These results were not considered to calculate the mean and the standard deviation.

In grey color are shaded the modes that we tried to optimize simultaneously in each configuration.

Table 10. Scaling Factors (EFDD) Using equation 3.

Mode	Numerical model	Masses attached in DOF									Mean	Std	Std/ mean (%)
		4 7	2 5	2 5 7	2 6 7	1 2 5 6	2 3 6 7	1 3 4 6 7	1 2 3 4 6	All DOF's			
1	0.5174	0.4697	0.4361	0.5213	0.4867	0.4492	0.5127	0.4842	0.2692	0.4935	0.4947	0.0191	3.87
2	0.5175	0.4905	0.5107	0.5001	0.4899	0.5079	0.5009	0.5061	0.5005	0.5113	0.5013	0.0081	1.61
3	0.5180	0.483	0.5086	0.4864	0.4816	0.4993	0.4951	0.5032	0.5008	0.4893	0.4944	0.0093	1.88
4	0.5196	0.4998	0.5241	0.4901	0.4863	0.4879	0.4710	0.4783	0.4914	0.4819	0.4852	0.0093	1.91
5	0.5228	0.5194	0.5825	0.582	0.4461	0.4600	0.4374	0.4717	0.4748	0.4729	0.4634	0.016	3.37

Table 11. Scaling Factors (SSI) Using equation 3.

Mode	Numerical model	Masses attached in DOF									Mean	Std	Std/ mean (%)
		4 7	2 5	2 5 7	2 6 7	1 2 5 6	2 3 6 7	1 3 4 6 7	1 2 3 4 6	All DOF's			
1	0.5174	0.4807	0.4592	0.4930	0.4836	0.4703	0.2793	0.4829	0.4405	0.4959	0.4872	0.0068	1.38
2	0.5175	0.4810	0.5071	0.4914	0.4864	0.4972	0.4936	0.5011	0.4991	0.5028	0.4953	0.0088	1.78
3	0.5180	0.4955	0.5056	0.4909	0.4840	0.5040	0.5038	0.5038	0.5084	0.4971	0.4991	0.0089	1.78
4	0.5196	0.4967	0.5309	0.4986	0.4874	0.4946	0.4764	0.4882	0.4973	0.4858	0.4895	0.0074	1.51
5	0.5228	0.4984	0.5743	0.5778	0.4435	0.4549	0.4403	0.4699	0.4721	0.4719	0.4618	0.014	3.03

It can be concluded that the modal parameters of the EFDD and SSI provide similar scaling factors (the difference being less than 3%). As it follows from the results, the empirical standard deviation is in the range 1.5%-5% for the first five modes.

It can also be observed that a mass change of 5% of the initial mass is not enough for all the configurations. The number and location of the masses have also to be considered.

The results obtained for the fifth mode shows that is difficult to achieve high frequency shifts when only few masses are attached to the structure. If possible, the number of masses should be equal or greater than the peaks and valleys of the shape of the higher mode.

It is not recommended trying to optimize the mass locations for many modes when only a few masses are going to be attached. Table 9 shows that the configuration with two masses attached in

DOF's 2 and 5, in which we tried to optimize the first six modes provides reasonable frequencies shifts only for two modes.

7 Conclusions

- Scaling factors have been estimated from a steel cantilever beam applying several mass change configurations ranging between 5% and 7% of the initial mass.
- An optimized mass change strategy was used to determine the best location for the masses. In addition to the mass magnitude, the effect of the number and location of the masses were studied.
- The scaling factors of the first five modes have estimated with a standard deviation of 1.5%-5%. However, new modal test and analysis were carried out to reduce the scatter in the estimated scaling factors corresponding to the first two modes.
- For the higher modes, it is difficult to achieve high frequency shifts using few masses. It is, therefore, recommended to consider a number of the attached masses equal or greater than the number of peaks and valleys of the mode shape

8 Acknowledgements

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